Nonlocality, Self-Adjointness and Θ-Vacuum in Quantum Field Theory in Spaces with Nontrivial Topology

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Abstract

We consider an analogue of the Aharonov-Bohm effect in quantum field theory: the fermionic vacuum attains nontrivial quantum numbers in the background of a magnetic vortex even in the case when the spatial region of nonvanishing external field strength is excluded. The dependence of the vacuum quantum numbers on the value of the vortex flux and the choice of the condition on the boundary of the excluded region is determined.

As is known, the singular static magnetic monopole background induces fermion number in the vacuum [1-3]

$$\langle N \rangle = -\frac{1}{\pi}\arctan\left(\tan\frac{\Theta}{2}\right), \tag{1}$$

where Θ is the parameter of a self-adjoint extension, which defines the boundary condition at a puncture corresponding to the location of the monopole; this results in the monopole becoming actually the dyon violating the Dirac quantization condition and CP symmetry.

In the present talk I shall be considering quantum numbers which are induced in the fermionic vacuum by the singular static magnetic string background. Since the deletion of a line , as compared to the deletion of a point, changes the topology of space in a much more essential way (fundamental group becomes nontrivial), the properties of the Θ-vacuum will appear to be much more diverse, as compared to Eq.(1). Restricting ourselves to a surface which is orthogonal to the string axis, let us consider 2+1-dimentional spinor electrodynamics on a plane with a puncture corresponding to the location of the string. I shall show that in this case the induced vacuum fermion number and magnetic flux depend on the self-adjoint extension parameter and the magnetic flux of the string as well.

The pertinent Dirac Hamiltonian has the form

$$H = -i\vec{\alpha}[\vec{\partial} - i\vec{V}(\vec{x})] + \beta m; \tag{2}$$

where $\vec{V}(\vec{x})$ is an external static vector potential. In a flat two-dimensional space ($\vec{x} = (x^1, x^2)$) the vacuum fermion number induced by such a background was calculated first in Ref.[4]

$$\langle N \rangle = -\frac{1}{2}\mathrm{sgn}(m)\Phi, \tag{3}$$

where $\mathrm{sgn}(u) = \left\{\begin{array}{rl} 1, & u > 0 \\ -1, & u < 0 \end{array}\right.$ is the sign function and $\Phi = \frac{1}{2\pi}\int d^2x B(\vec{x})$ is the total flux (in the units of 2π) of the external magnetic field strength $B(\vec{x}) = \vec{\partial} \times \vec{V}(\vec{x})$ piercing the two-dimensional space (plane); note that the mass parameter m in Eq.(2) can take both positive and negative values in two and any even number of spatial dimensions.

It should be emphasized, however, that Eq.(3) is valid for regular external field configurations only, i.e. $B(\vec{x}) = B_{\mathrm{reg}}(\vec{x})$, where $B_{\mathrm{reg}}(\vec{x})$ is a continuous in the whole function that can grow at most as $O(|\vec{x} - \vec{x}_s|^{-2+\varepsilon})$ ($\varepsilon > 0$) at separate points; as to a vector potential $\vec{V}(\vec{x}) = (V_1(\vec{x}), V_2(\vec{x}))$, it is unambiguously defined everywhere on the plane. The regular configuration of an external field polarizes the vacuum

locally, and Eq.(3) is just the integrated version of the linear relation between the vacuum fermion number density and the magnetic field strength.

One can ask the following question: whether the nonlocal effects of the external field background are possible, i.e., if the spatial region of nonvanishing field strength is excluded, whether there will be vacuum polarization in the remaining part of space? For the positive answer it is necessary, although not sufficient, that the latter spatial region be of nontrivial topology [5] (see also Ref.[6]). However, the condition on the boundary of the excluded region has not been completely specified. In the present talk this point will be clarified by considering the whole set of boundary conditions which are compatible with the self-adjointness of the Dirac Hamiltonian in the remaining region.

We shall be interested in the situation when the volume of the excluded region is shrinked to zero, while the global characteristics of the external field in the excluded region is retained nonvanishing. This implies that singular, as well as regular, configurations of external fields have to be considered. In particular, in two spatial dimensions the magnetic field strength is taken to be a distribution (generalized function)

$$B(\vec{x}) = B_{\rm reg}(\vec{x}) + 2\pi\Phi^{(0)}\delta(\vec{x}), \tag{4}$$

where $\Phi^{(0)}$ is the total magnetic flux (in the units of 2π) in the excluded region which is placed at the origin $\vec{x}=0$. As to the vector potential, it is unambiguously defined everywhere with the exception of the origin, i.e. the limiting value $\lim_{|\vec{x}|\to 0}\vec{V}(\vec{x})$ does not exist, or, to be more precise, a singular magnetic vortex is located at the origin

$$\lim_{|\vec{x}|\to 0} \vec{x}\times\vec{V}(\vec{x}) = \Phi^{(0)}. \tag{5}$$

Certainly, a plane has trivial topology, $\pi_1 = 0$, while a plane with a puncture where the vortex is located has nontrivial topology, $\pi_1 = {\rm Z}$; here Z is the set of integer numbers and π_1 is the first homotopy group of the surface.

The total magnetic flux through the punctured plane is obviously defined as

$$\Phi = \frac{1}{2\pi}\int d^2x B_{\rm reg}(\vec{x}) = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi[\vec{x}\times\vec{V}(\vec{x})]\Big|_{r=0}^{r=\infty}, \tag{6}$$

where the polar coordinates $r=|\vec{x}|$ and $\varphi = \arctan(x^2/x^1)$ are introduced.

The Dirac equation with the Hamiltonian (2) on a punctured plane is invariant with respect to the gauge transformations

$$G: \vec{V}(\vec{x}) \to \vec{V}(\vec{x}) + \vec{\partial}\Lambda(\vec{x}), \quad \psi(\vec{x}) \to {\rm e}^{i\Lambda(\vec{x})}\psi(\vec{x}). \tag{7}$$

Although the vector potential in any gauge is single-valued on a punctured plane, this is not true for the gauge function $\Lambda(\vec{x})$. Since the magnetic flux Φ(6) (and the field strength $B_{\rm reg}(\vec{x})$) remains invariant under gauge transformations, the most general condition on $\Lambda(\vec{x})$ takes the form

$$\Lambda(r, \varphi + 2\pi) = \Lambda(r, \varphi) + 2\pi\Upsilon_\Lambda, \tag{8}$$

where Υ_Λ is the independent of r and φ parameter of the gauge transformation; incidentally the magnetic flux of the vortex $\Phi^{(0)}$(5) is changed: $\Phi^{(0)} \to \Phi^{(0)} + \Upsilon_\Lambda$. If one takes a single-valued wave function, $\psi(r, \varphi + 2\pi) = \psi(r, \varphi)$, then, after applying a gauge transformation to it, one gets a wave function satisfying the condition $({\rm e}^{i\Lambda}\psi)(r, \varphi+2\pi) = {\rm e}^{i2\pi\Upsilon_\Lambda}({\rm e}^{i\Lambda}\psi)(r, \varphi)$. Thus the set of wave functions on a punctured plane is much richer than that of wave functions on a plane without a puncture (in the latter case only the gauge transformations with $\Upsilon_\Lambda = 0$ are admissible). Certainly, there are no reasons to impose the condition of single-valuedness on the initial function, and in the most general case one takes

$$\psi(r, \varphi + 2\pi) = {\rm e}^{i2\pi\Upsilon}\psi(r, \varphi), \tag{9}$$

and after applying a gauge transformation one gets

$$({\rm e}^{i\Lambda}\psi)(r, \varphi + 2\pi) = {\rm e}^{i2\pi(\Upsilon+\Upsilon_\Lambda)}({\rm e}^{i\Lambda}\psi)(r, \varphi). \tag{10}$$

Therefore, if one admits singular gauge transformations ($\Upsilon_\Lambda \neq 0$), as well as regular ones ($\Upsilon_\Lambda = 0$), then one has to consider wave functions defined on a plane with a cut which starts from the puncture and goes to infinity in the radial direction at, say, the angle $\varphi = \varphi_c$. The boundary conditions on the sides of the cut are globally parametrized by the values of Υ.

All this can be presented in a more refined way, using the notion of a self-adjoint extension of a Hermitian (symmetric) operator. The orbital angular momentum operator, $-i\partial_\varphi$, entering the Dirac Hamiltonian(2) is Hermitian, but not self-adjoint, when defined on the domain of functions satisfying, say, $\psi(r, \varphi_c+2\pi) = \psi(r, \varphi_c) = 0$; this operator has the deficiency index equal to (1,1). The use of the Weyl–von Neumann theory of self-adjoint extension [7] yields that $-i\partial_\varphi$ becomes self-adjoint, when defined on the domain of functions satisfying Eq.(9) with $\varphi = \varphi_c$, where the values of Υ parametrize the family of extensions. It should be stressed that Υ, as well as $\Phi^{(0)}$, is changed under the singular gauge transformations (compare Eqs. (9) and (10)), while the difference $\Phi^{(0)} - \Upsilon$ remains invariant.

Let us turn now to the boundary condition at the puncture $\vec{x} = 0$. In the following our concern will be in the case in which the regular part of the magnetic field is absent, $B_{\rm reg}(\vec{x}) = 0$. Then, in the representation with $\alpha_1 = \sigma_1$, $\alpha_2 = \sigma_2$ and

$\beta = \sigma_3$ (σ_j are the Pauli matrices) the spinor wave function satisfying the Dirac equation and the condition (9) has the form

$$\psi(\vec{x}) = \sum_{n\in \mathrm{Z}} \begin{pmatrix} f_n(r)\exp[i(n+\Upsilon)\varphi] \\ g_n(r)\exp[i(n+1+\Upsilon)\varphi] \end{pmatrix}, \tag{11}$$

where the radial functions, in general, are

$$\begin{pmatrix} f_n(r) \\ g_n(r) \end{pmatrix} = \begin{pmatrix} C_n^{(1)}(E) J_{n-\Phi^{(0)}+\Upsilon}(kr) + C_n^{(2)}(E) Y_{n-\Phi^{(0)}+\Upsilon}(kr) \\ \frac{ik}{E+m}[C_n^{(1)}(E) J_{n+1-\Phi^{(0)}+\Upsilon}(kr) + C_n^{(2)}(E) Y_{n+1-\Phi^{(0)}+\Upsilon}(kr)] \end{pmatrix}, \tag{12}$$

$k = \sqrt{E^2 - m^2}$, $J_\mu(z)$ and $Y_\mu(z)$ are the Bessel and the Neumann functions of the order μ. It is clear that the condition of regularity at $r = 0$ can be imposed on both f_n and g_n for all n in the case of integer values of the quantity $\Phi^{(0)} - \Upsilon$ only. Otherwise, the condition of regularity at $r = 0$ can be imposed on both f_n and g_n for all but $n = n_0$, where

$$n_0 = [\![\Phi^{(0)} - \Upsilon]\!] \,, \tag{13}$$

$[\![u]\!]$ is the integer part of the quantity u (i.e. the integer which is less than or equal to u); in this case at least one of the functions, f_{n_0} or g_{n_0}, remains irregular, although square integrable, with the asymptotics r^{-p} ($p < 1$) at $r \to 0$ [8]. The question arises then, what boundary condition, instead of regularity, is to be imposed on f_{n_0} and g_{n_0} at $r = 0$ in the latter case?

To answer this question, one has to find the self-adjoint extension for the partial Hamiltonian corresponding to the mode with $n = n_0$. If this Hamiltonian is defined on the domain of regular at $r = 0$ functions, then it is Hermitian, but not self-adjoint, having the deficiency index equal to (1,1). Hence the family of self-adjoint extensions is labeled by one real continuous parameter denoted in the following by Θ. It can be shown (see Ref.[9]) that, for the partial Hamiltonian to be self-adjoint, it has to be defined on the domain of functions satisfying the boundary condition

$$\lim_{r\to 0} \cos\left(\frac{\Theta}{2} + \frac{\pi}{4}\right)\left(|m|r\right)^F f_{n_0}(r) = i \lim_{r\to 0} \sin\left(\frac{\Theta}{2} + \frac{\pi}{4}\right)\left(|m|r\right)^{1-F} g_{n_0}(r), \tag{14}$$

where

$$F = \{\!\!\{\Phi^{(0)} - \Upsilon\}\!\!\}, \tag{15}$$

$\{\!\!\{u\}\!\!\}$ is the fractional part of the quantity u, $\{\!\!\{u\}\!\!\} = u - [\![u]\!]$, $0 \le \{\!\!\{u\}\!\!\} < 1$; note here that Eq.(14) implies that $0 < F < 1$, since in the case of $F = 0$ both f_{n_0} and g_{n_0} satisfy the condition of regularity at $r = 0$.

Using the explicit form of the solution to the Dirac equation in the background of a singular magnetic vortex, it is straightforward to calculate the vacuum fermion

number induced on a punctured plane. As follows already from the preceding discussion, the vacuum fermion number vanishes in the case of integer values of $\Phi^{(0)} - \Upsilon$ ($F = 0$), since this case is indistinguishable from the case of the trivial background, $\Phi^{(0)} = \Upsilon = 0$. In the case of noninteger values of $\Phi^{(0)} - \Upsilon$ ($0 < F < 1$) we get (details will be published elsewhere)

$$\langle N \rangle = \begin{cases} -\frac{1}{2}\mathrm{sgn}(m)F \,, & \Theta = \frac{\pi}{2}(\mathrm{mod} 2\pi) \\ \frac{1}{2}\mathrm{sgn}(m)\,(1-F)\ , & \Theta = -\frac{\pi}{2}(\mathrm{mod} 2\pi) \end{cases} \tag{16}$$

and

$$\langle N \rangle = -\frac{1}{2}\mathrm{sgn}(m)\Big(F - \frac{1}{2}\Big) - \frac{1}{4\pi}\int\limits_1^\infty \frac{dv}{v\sqrt{v-1}} \times$$
$$\times \frac{\mathrm{sgn}(m)(Av^F - A^{-1}v^{1-F}) + 4(F-\frac{1}{2})(v-1)}{Av^F + 2\mathrm{sgn}(m) + A^{-1}v^{1-F}}, \quad \Theta \neq \frac{\pi}{2}(\mathrm{mod}\pi), \tag{17}$$

where

$$A = 2^{1-2F}\frac{\Gamma(1-F)}{\Gamma(F)}\tan\Big(\frac{\Theta}{2} + \frac{\pi}{4}\Big), \tag{18}$$

$\Gamma(u)$ is the Euler gamma-function. Eqs.(16) and (17) can be presented in another form

$$\langle N \rangle = \left\{ \begin{array}{ll} -\frac{1}{2}\mathrm{sgn}(m)F + & \\ +\frac{1}{2}\mathrm{sgn}\,[A + \mathrm{sgn}(m)]\,, & \Theta \neq \frac{\pi}{2}(\mathrm{mod}2\pi), \\ -\frac{1}{2}\mathrm{sgn}(m)F, & \Theta = \frac{\pi}{2}(\mathrm{mod}2\pi), \end{array} \right\}, \qquad 0 < F < \frac{1}{2}\,, \tag{19}$$

$$\langle N \rangle = -\frac{1}{\pi}\arctan\Big\{\tan\Big[\frac{\Theta}{2} + \frac{\pi}{4}(1 - \mathrm{sgn}(m))\Big]\Big\} \quad , \qquad F = \frac{1}{2}\,, \tag{20}$$

$$\langle N \rangle = \left\{ \begin{array}{ll} \frac{1}{2}\mathrm{sgn}(m)\,(1-F) - & \\ -\frac{1}{2}\mathrm{sgn}\,[A^{-1} + \mathrm{sgn}(m)]\,, & \Theta \neq -\frac{\pi}{2}(\mathrm{mod}2\pi), \\ \frac{1}{2}\mathrm{sgn}(m)\,(1-F)\,, & \Theta = -\frac{\pi}{2}(\mathrm{mod}2\pi), \end{array} \right\}, \ \frac{1}{2} < F < 1\,; \tag{21}$$

note that Eq.(20) in the case of $m > 0$, when it coincides with Eq.(1), has been obtained earlier in Ref.[10]. We get also the relations

$$\lim_{F\to 0}\langle N\rangle = \frac{1}{2}\mathrm{sgn}(m), \quad \Theta \neq \frac{\pi}{2}(\mathrm{mod}2\pi) \tag{22}$$

and

$$\lim_{F\to 1}\langle N\rangle = -\frac{1}{2}\mathrm{sgn}(m), \quad \Theta \neq -\frac{\pi}{2}(\mathrm{mod}2\pi), \tag{23}$$

indicating that the vacuum fermion number is not, in general, continuous at integer values of $\Phi^{(0)} - \Upsilon$; the limiting values (22) and (23) differ from the value at $F = 0$ exactly, the latter being equal, as noted before, to zero.

It is obvious that the vacuum fermion number at fixed values of Υ and Θ is periodic in the value of $\Phi^{(0)}$. This feature (periodicity in $\Phi^{(0)}$) is also shared by the quantum-mechanical scattering of a nonrelativistic particle in the background of a singular magnetic vortex, known as the Aharonov-Bohm effect [11]. Since there appear assertions in the literature which deny the periodicity of the vacuum fermion number in $\Phi^{(0)}$ [12,13], the following comments on the result (19)–(21) will be clarifying.

Under the charge conjugation,

$$C: \quad \vec{V} \to -\vec{V}, \quad \psi \to \sigma_1\psi^*, \quad \Upsilon \to -\Upsilon, \tag{24}$$

the fermion number operator and its vacuum value are to be odd, $N \to -N$ and $\langle N\rangle \to -\langle N\rangle$. Evidently, the result (19)–(21) is not, since the boundary condition (14) breaks, in general, the charge conjugation symmetry. However, for certain choices of the parameter Θ this symmetry can be retained [14].

In particular, choosing

$$\left.\begin{array}{ll} \Theta = \frac{\pi}{2}(\mathrm{mod}2\pi), & \Phi^{(0)} - \Upsilon > 0 \\ \Theta = -\frac{\pi}{2}(\mathrm{mod}2\pi), & \Phi^{(0)} - \Upsilon < 0 \end{array}\right\} \quad (\Phi^{(0)} - \Upsilon \neq n, \quad n \in \mathrm{Z}), \tag{25}$$

which corresponds to the boundary condition of Refs.[15,16], one obtains [12, 13, 17]

$$\langle N\rangle = \left\{\begin{array}{ll} -\frac{1}{2}\mathrm{sgn}(m)F, & \Phi^{(0)} - \Upsilon > 0 \\ \frac{1}{2}\mathrm{sgn}(m)(1-F), & \Phi^{(0)} - \Upsilon < 0 \end{array}\right\}, \quad 0 < F < 1, \tag{26}$$

which is odd under the charge conjugation but is not periodic in $\Phi^{(0)}$.

No wonder that there exists a choice of Θ respecting both the periodicity in $\Phi^{(0)}$ and the charge conjugation symmetry, namely,

$$\begin{array}{ll} \Theta = \frac{\pi}{2}(\mathrm{mod}2\pi), & 0 < F < \frac{1}{2} \\ \Theta = -\frac{\pi}{2}[1 - \mathrm{sgn}(m)](\mathrm{mod}2\pi), & F = \frac{1}{2} \\ \Theta = -\frac{\pi}{2}(\mathrm{mod}2\pi), & \frac{1}{2} < F < 1 \end{array}, \tag{27}$$

which corresponds to the condition of minimal irregularity, i.e. to the radial functions being divergent at $r \to 0$ at most as r^{-p} with $p \leq \frac{1}{2}$. This is the boundary condition, with the use of which the result of Ref.[5] is obtained:

$$\langle N \rangle = \frac{1}{2}\mathrm{sgn}(m)\Big[\frac{1}{2}\mathrm{sgn}_0\Big(F - \frac{1}{2}\Big) - F + \frac{1}{2}\Big], \tag{28}$$

where $\mathrm{sgn}_0(u) = \begin{cases} \mathrm{sgn}(u), & u \neq 0 \\ 0, & u = 0 \end{cases}$. Note that Eq.(28) is continuous at integer values of $\Phi^{(0)} - \Upsilon$ and discontinuous at half-integer ones.

Another choice compatible with the periodicity in $\Phi^{(0)}$ and the symmetry (24) is $\Theta = -\frac{\pi}{2}[1 - \mathrm{sgn}(m)](\mathrm{mod}2\pi)$ for $0 < F < 1$; then the vacuum fermion number is discontinuous both at integer and half-integer values of $\Phi^{(0)} - \Upsilon$.

We have calculated also the total magnetic flux induced in the fermionic vacuum on a punctured plane

$$\Phi^{(I)} = -\frac{e^2 F(1-F)}{2\pi|m|}\left[\frac{1}{6}\Big(F - \frac{1}{2}\Big) + \frac{1}{4\pi}\int\limits_1^\infty \frac{dv}{v\sqrt{v-1}}\frac{Av^F - A^{-1}v^{1-F}}{Av^F + 2\mathrm{sgn}(m) + A^{-1}v^{1-F}}\right]; \tag{29}$$

note that the coupling constant e relating the vacuum current to the vacuum magnetic field strength (via the Maxwell equation) has the dimension $\sqrt{|m|}$ in 2 + 1-dimensional space-time. At half-integer values of $\Phi^{(0)} - \Upsilon$ we get

$$\Phi^{(I)} = -\frac{e^2}{8\pi^2 m}\arctan\Big\{\tan\Big[\frac{\Theta}{2} + \frac{\pi}{4}(1 - \mathrm{sgn}(m))\Big]\Big\} \qquad \Big(F = \frac{1}{2}\Big). \tag{30}$$

The vacuum magnetic flux under the boundary condition (25) is given in Ref.[12]. Under the boundary condition (27) we get

$$\Phi^{(I)} = \frac{e^2 F(1-F)}{12\pi|m|}\Big[\frac{3}{2}\mathrm{sgn}_0(F - \frac{1}{2}) - F + \frac{1}{2}\Big], \tag{31}$$

which is both periodic in $\Phi^{(0)}$ and C-odd. As it follows from Eq.(31), the vacuum under the boundary condition (27) is in a certain sense of a diamagnetic type at $0 < F < \frac{1}{2}$ and of a paramagnetic type at $\frac{1}{2} < F < 1$.

In conclusion, we present the evident relations

$$\sum_{\mathrm{sgn}(m)} \langle N \rangle = \left\{\begin{array}{ll} \frac{1}{2}\mathrm{sgn}\,(A+1) + & \\ +\frac{1}{2}\mathrm{sgn}\,(A-1)\ , & \Theta \neq \frac{\pi}{2}(\mathrm{mod}2\pi), \\ 0, & \Theta = \frac{\pi}{2}(\mathrm{mod}2\pi), \end{array}\right\},\ \ 0 < F < \frac{1}{2}\ , \tag{32}$$

$$\sum_{\text{sgn}(m)} \langle N \rangle = -\frac{2}{\pi}\arctan\left(\tan\frac{\Theta}{2}\right) + \frac{1}{2}\text{sgn}\left(\tan\frac{\Theta}{2}\right) \quad , \qquad F = \frac{1}{2}, \tag{33}$$

$$\sum_{\text{sgn}(m)} \langle N \rangle = \left\{ \begin{array}{ll} -\frac{1}{2}\text{sgn}\,(A^{-1}+1) - & \\ -\frac{1}{2}\text{sgn}\,(A^{-1}-1)\ , & \Theta \neq -\frac{\pi}{2}(\text{mod}2\pi), \\ 0, & \Theta = -\frac{\pi}{2}(\text{mod}2\pi), \end{array} \right\}, \quad \frac{1}{2} < F < 1 \tag{34}$$

and

$$\sum_{\text{sgn}(m)} \Phi^{(I)} = -\frac{e^2 F(1-F)}{2\pi|m|}\left[\frac{1}{3}\left(F-\frac{1}{2}\right) + \right.$$
$$\left. + \frac{1}{2\pi}\int\limits_1^\infty \frac{dv}{v\sqrt{v-1}} \frac{A^2 v^{2F} - A^{-2} v^{2(1-F)}}{A^2 v^{2F} + A^{-2} v^{2(1-F)} + 2v - 4}\right], \tag{35}$$

which are in contrast with the fact that summation of Eq.(3) over sgn(m) yields zero.

Thus, we see that quantum numbers induced by a singular magnetic vortex in the fermionic vacuum depend on the gauge invariant quantities, $\Phi^{(0)} - \Upsilon$ and Θ. For certain choices of Θ the vacuum quantum numbers are periodic in $\Phi^{(0)} - \Upsilon$ and have definite C-parity.

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[17] It is amusing to note that Eq.(26) has been obtained by the authors of Ref.[12] at the intermediate stage of calculations. However, then these authors add a phantom contribution to the correct expression and get the incorrect one as a final result; the latter looks like Eq.(3) which is irrelevant to the case considered. As to the authors of Ref.[13], who have obtained also Eq.(26), they, in addition, identify erroneously the limiting value of this expression at $F \to 1$ with the value of $\langle N \rangle$ at $\Phi^{(0)} - \Upsilon = 1$. (Note that the authors of both Refs.[12,13] consider the case of $\Upsilon = 0$).